

Model Documentation of the Acrobot

1 Nomenclature

1.1 Nomenclature for Model Equations

s_i	center of gravity distance of the bar for $i = 1, 2$
m_i	mass of the bar for $i = 1, 2$
J_i	moment of inertia of the bar for $i = 1, 2$
l_1	length of the first bar
g	acceleration due to gravity
p_1	angle between the vertically downwards rest position and the first bar
q_1	angle between the first and the second bar
\dot{p}_1	angle velocity of the first bar
\dot{q}_1	angle velocity of the second bar
τ_1	input force at the joint

1.2 Graphic of the Structure

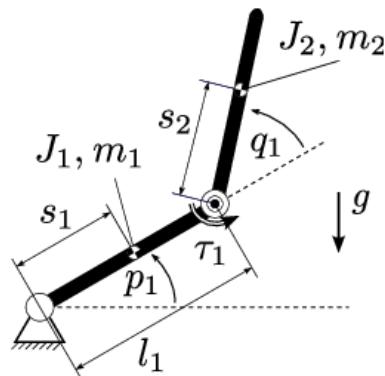


Figure 1: Acrobot

Source: Knoll, Carsten/Acrobot (=unteraktuierter Zweigelenkmanipulator, Stellglied im Ellenbogengelenk)

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (p_1 \ q_1 \ \dot{p}_1 \ \dot{q}_1)^T & &= (x_1 \ x_2 \ x_3 \ x_4)^T \\ \underline{u} &= \tau_1 & &= u_1 \end{aligned}$$

Kinetic Energy:

$$\begin{aligned}
T = & \frac{J_1 x_3^2}{2} + \frac{J_2 (x_3 + x_4)^2}{2} + \frac{m_1 x_3^2 s_1^2 \sin(x_1)^2}{2} + \frac{m_1 x_3^2 s_1^2 \cos(x_1)^2}{2} \\
& + m_2 (-l_1 x_3 \sin(x_1) - \frac{s_2 (x_3 + x_4) \sin(x_1 + x_2)}{2})^2 + m_2 (l_1 x_3 \cos(x_1) \\
& + \frac{s_2 (x_3 + x_4) \cos(x_1 + x_2)}{2})^2
\end{aligned}$$

Potential Energy:

$$V = gm_1 s_1 \sin(x_1) + gm_2 (l_1 \sin(x_1) + s_2 \sin(x_1 + x_2))$$

Parameters: $s_1, s_2, m_1, m_2, J_1, J_2, l_1, g$

Outputs: \underline{x}

2.1 Assumptions

1. The rest position of the acrobot is vertically downward.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of first bar	s_1	0.25	m
center of gravity distance of second bar	s_2	0.25	m
mass of first bar	m_1	1	kg
mass of second bar	m_2	1	kg
moment of inertia of first bar	J_1	0.006667	$kg \cdot m^2$
moment of inertia of second bar	J_2	0.01333	$kg \cdot m^2$
length of first bar	l_1	0.5	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

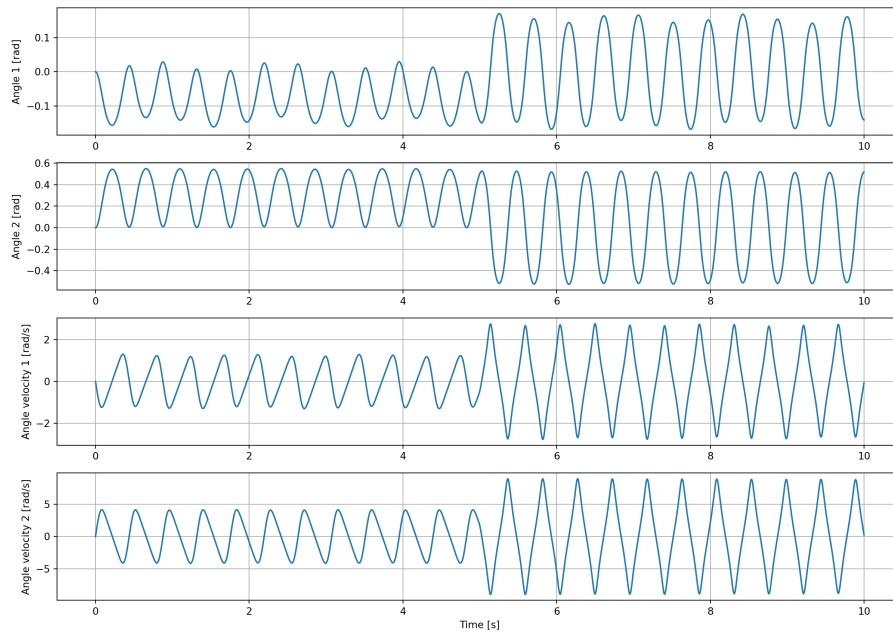


Figure 2: Simulation of the acrobot.

5 Simulation

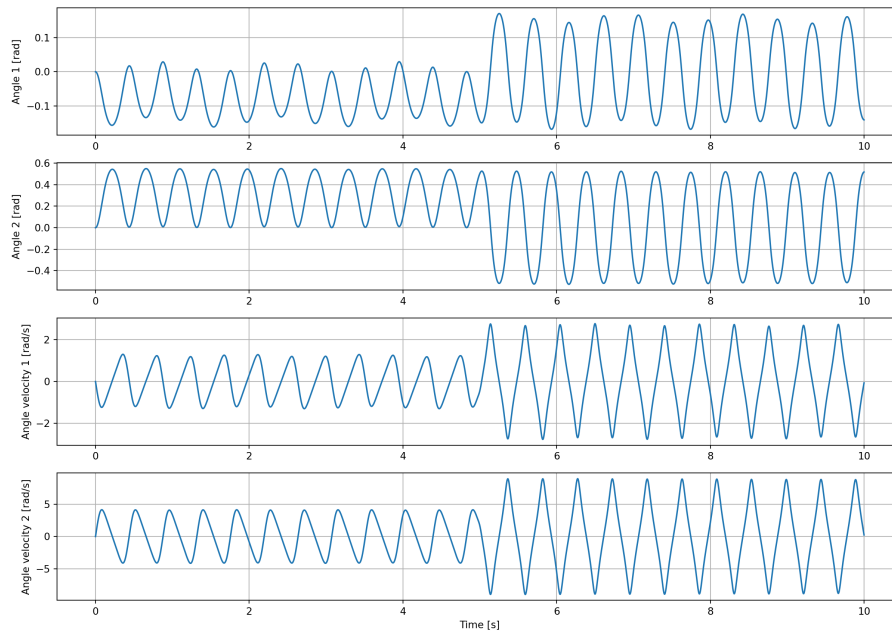


Figure 3: Simulation of the acrobot.

References

- [1] Knoll, Carsten: *Acrobot (=unteraktuierter Zweigelenkmanipulator, Stellglied im Ellenbogengelenk)*, Jupyter Notebook published 2017.
https://github.com/cknoll/beispiele/blob/master/acrobot_rwa.ipynb