Model Documentation of the Ball and Beam

1 Nomenclature

1.1 Nomenclature for Model Equations

m_1	mass of the ball
J_1	moment of inertia of the beam
J_2	moment of inertia of the ball
r	radius of the ball
g	acceleration due to gravity
τ	torque in the middle of the beam
q_1	distance between the ball and the middle of the beam
q_2	rotation angle of the beam

1.2 Graphic of the Structure

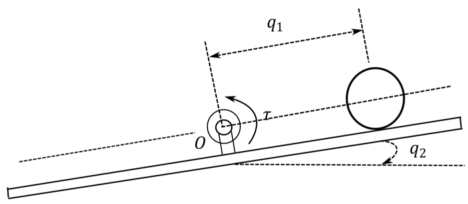

Figure 1: Structure of the Ball and Beam
Source: Wang, Yang/Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2)^T = (x_1 \ x_2 \ x_3 \ x_4)^T$$
$$u = \tau$$

System Equations:

$$\dot{x}_1 = x_3 \tag{1a}$$
$$\dot{x}_2 = x_4 \tag{1b}$$
$$\dot{x}_3 = \frac{m_1 x_1 x_4^2 - g m_1 \sin x_2}{m_1 + \frac{J_2}{r^2}} \tag{1c}$$
$$\dot{x}_4 = \frac{u - m_1 g x_1 \cos x_2 - 2 m_1 x_1 x_3 x_4}{J_1 + J_2 + m_1 x_1^2} \tag{1d}$$

Parameters: m_1, J_1, J_2, r, g
Outputs: $\underline{x}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the ball	m_1	0.05	kg
moment of inertia of the beam	J_1	0.02	$kg \cdot m^2$
moment of inertia of the ball	J_2	$2.0 \cdot 10^{-6}$	$kg \cdot m^2$
radius of the ball	r	0.01	m
acceleration due to gravity	g	10	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.
Rotational Energy:

$$T_{rball} = \frac{1}{2}J_2 x_4^2 + \frac{1}{2}\frac{J_2}{r^2}x_3^2 \tag{2}$$

$$T_{rbeam} = \frac{1}{2}J_1 x_4^2 \tag{3}$$

Translational Energy:

$$T_t = \frac{1}{2}m_1(x_3^2 + x_1^2 x_4^2) \tag{4}$$

Potential Energy:

$$V = m_1 g x_1 \sin x_2 \tag{5}$$

The depicted open loop control in figure 2 diverges as expected.

4 Simulation

5 Simulation

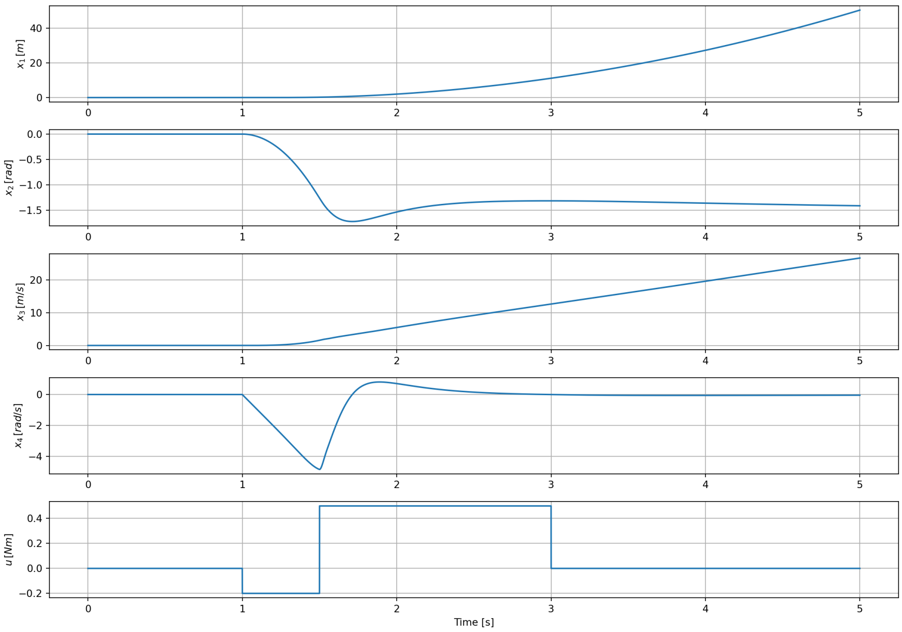

Figure 2: Simulation of the ball beam.

References

[1] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)

[2] J. Hauser, S. Sastry and P. Kokotovic *Nonlinear control via approximate input-output linearization: the ball and beam example.* In: Decision and Control, 1989, Proceedings of the 28th IEEE Conference on, S. 1987–1993 vol.3, Dec 1989.