Model Documentation of the Ball in the Tube

1 Nomenclature

1.1 Nomenclature for Model Equations

A_B	ball cross-sectional area
A_{SP}	air gap cross-sectional area
m	mass of the styrofoam ball
g	acceleration due to gravitation
T_M	time constant of the motor model
k_M	amplification of the motor model
k_V	proportional factor between the volume flow of the fan and the absolute speed rotation
k_L	parameter, describes the relation between the air gap velocity and the flow resistance force
η	rotation speed
η_0	basic rotation speed
h	height of the ball
$\dot{h}$	velocity of the ball
u_{PWM}	motor control voltage

1.2 Graphic of the Structure

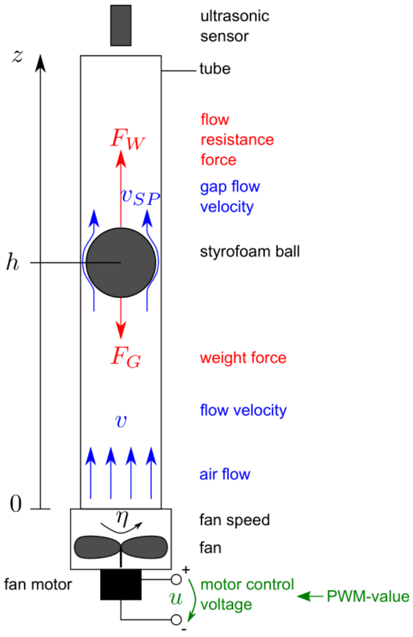

Figure 1: Model Structure.
Source: Institut of Control Theory TU Dresden: Regelungstechnikpraktikum, Praktikumsanleitung

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (\eta \ h \ \dot{h})^T &&= (x_1 \ x_2 \ x_3)^T \\ \underline{u} &= u_{PWM} &&= u \end{aligned}$$

System Equations:

$$\dot{x}_1 = -\frac{1}{T_M} x_1 + \frac{k_M}{T_M} u \tag{1a}$$

$$\dot{x}_2 = x_3 \tag{1b}$$

$$\dot{x}_3 = \frac{k_L}{m} \left(\frac{k_V (x_1 + \eta_0) - A_B \dot{h}}{A_{SP}} \right)^2 - g \tag{1c}$$

Parameters: A_B, A_{SP}, m, g, T_M, k_M, k_V, k_L, n_0
Outputs: h

2.1 Assumptions

1. The behavior of the ball does not influence the air flow provided by the fan.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
ball cross-sectional area	A_B	0.002827	m^2
air gap cross-sectional area	A_{SP}	0.0004299	m^2
mass of the ball	m	0.0028	kg
acceleration due to gravitation	g	9.81	$\frac{m}{s^2}$
time constant	T_M	0.369	s
amplification	k_M	0.273	s^{-1}
proportional factor	k_V	0.00012	m^3
parameter	k_L	0.0002823	$\frac{kg}{m}$
basic rotation speed	n_0	456	$\frac{U}{min}$

3 Derivation and Explanation

The model of the system consists of two submodels. The first one is the model of the ball. Essentially there two forces acting on the ball, the weight force F_g and the force exerted by the flow resistance F_w.

$$F_g = mg \tag{2}$$

$$F_w = \frac{1}{2} c_w \varrho_L A_B v_{SP}^2. \tag{3}$$

After inserting the equation for the relative velocity between the sphere and the medium flowing around it v_{SP} and using the proportinal factor k_V we get

$$F_w = \frac{1}{2} c_w \varrho_L A_B \left(\frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_S P} \right)^2 . \tag{4}$$

If we now introduce the parameter k_L

$$F_w = k_L \left(\frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_S P} \right)^2 \tag{5}$$

results. Using the vertical force balance, the following equation can be set up.

$$m\ddot{h} = k_L \left(\frac{k_V(\eta + \eta_0) - A_B \dot{h}}{A_S P} \right)^2 - mg \tag{6}$$

The second model is the fan. We assume an ordinary fan with pulse width modulation (PWM) control. We are using a model with PT_1-behaviour with the time constant T_M an the amplification k_M.

$$T_M \dot{\eta} + \eta = k_M u_{PWM} \tag{7}$$

4 Simulation

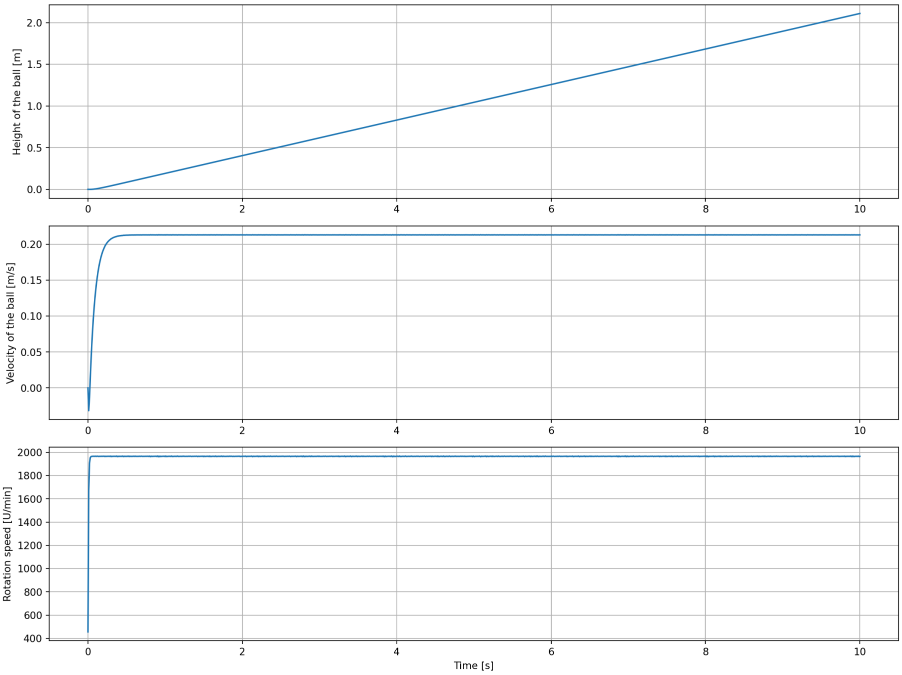

Figure 2: Simulation of the ball in tube.

References

[1] Institut of Control Theory TU Dresden: *Regelungstechnikpraktikum, Praktikumsanleitung*, published on OPAL April 2022.
(not publicly accessible)