Model Documentation of the Overhead Crane

1 Nomenclature

1.1 Nomenclature for Model Equations

s_1	center of gravity distance of the load
m_1	mass of trolley 1
m_2	mass of load
m_3	mass of trolley 2
J_2	moment of inertia of the load
l_0	initial distance between the trolleys
l_i	length of rope i, where $i = 1, 2$
g	acceleration due to gravity
p_1	absolute x position of the load
p_2	absolute y position of the load
p_3	angle between basis and the load
q_i	displacement of the trolley i in x direction, where $i = 1, 2$

1.2 Graphic of the Structure

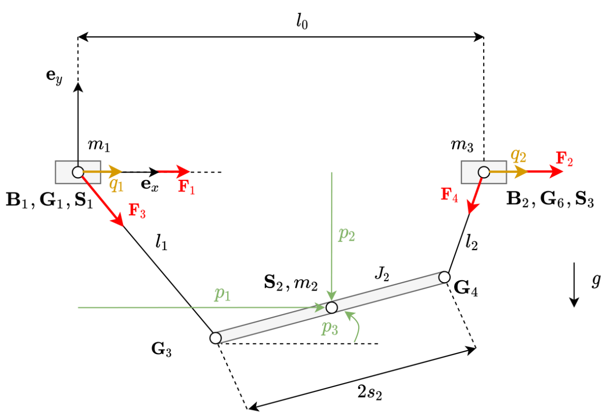

Figure 1: Structure of the Overhead Crane.
Source: Wrede, Konstantin / Modellbildung und Reglerentwurf für ein Brückenkransystem

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (p_1\ p_2\ p_3\ q_1\ q_2\ \dot{p}_1\ \dot{p}_2\ \dot{p}_3\ \dot{q}_1\ \dot{q}_2)^T$$
$$= (x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10})^T$$
$$\underline{u} = (u_1\ u_2\ u_3\ u_4)^T$$

System Equations:

$$0 = m_2\ddot{x}_1 - \frac{u_4(-l_0 + s_2\cos(x_3) + x_1 - x_5)}{l_2} - \frac{u_3(-s_2\cos(x_3) + x_1 - x_4)}{l_1} \tag{1a}$$

$$0 = gm_2 + m_2\ddot{x}_2 - \frac{u_4(s_2\sin(x_3) + x_2)}{l_2} - \frac{u_3(-s_2\sin(x_3) + x_2)}{l_1} \tag{1b}$$

$$0 = J_2\ddot{x}_3 - \frac{s_2u_4(s_2\sin(x_3) + x_2)\cos(x_3)}{l_2} + \frac{s_2u_4(-l_0 + s_2\cos(x_3) + x_1 - x_5)\sin(x_3)}{l_2} + \frac{s_2u_3(-s_2\sin(x_3) + x_2)\cos(x_3)}{l_1} - \frac{s_2u_3(-s_2\cos(x_3) + x_1 - x_4)\sin(x_3)}{l_1} \tag{1c}$$

$$0 = m_1\ddot{x}_4 - u_1 + \frac{u_3(-s_2\cos(x_3) + x_1 - x_4)}{l_1} \tag{1d}$$

$$0 = m_3\ddot{x}_5 - u_2 + \frac{u_4(-l_0 + s_2\cos(x_3) + x_1 - x_5)}{l_2} \tag{1e}$$

Parameters: $s_1, m_1, m_2, m_3, J_2, l_0, l_1, l_2$
Outputs: $\underline{\mathrm{x}}$

2.1 Assumptions

1. The movement of all components is only considered in the vertical plane.
2. The ropes are assumed to be massless.
3. The load is considered to have a homogeneous mass distribution.
4. Dissipative forces are not taken into account.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
center of gravity distance of the load	s_2	0.15	m
mass of trolley 1	m_1	0.45	kg
mass of load	m_2	0.557	kg
mass of trolley 2	m_3	0.45	kg
moment of inertia of the load	J_2	0.000221	$kg \cdot m^2$
initial distance between the trolleys	l_0	0.5	m
length of rope 1	l_1	0.4	m
length of rope 2	l_2	0.3	m
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

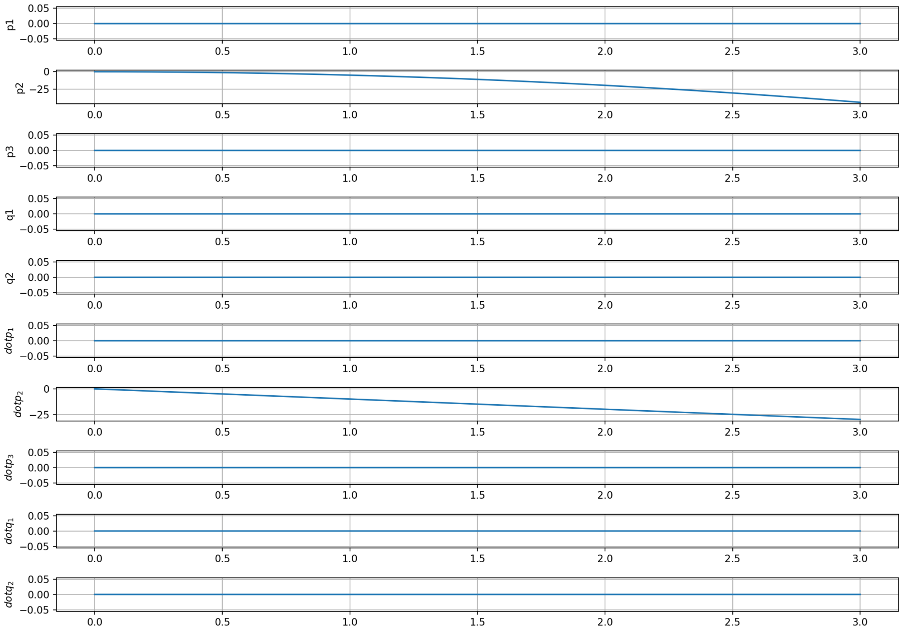

Figure 2: Simulation of the double crane.

References

[1] Wrede, Konstantin: *Modellbildung und Reglerentwurf für ein Brückenkransystem*, student research project at the Institut of Control Theory TU Dresden, published 2022.
(not publicly accessible)