Model Documentation of the Furuta Pendulum

1 Nomenclature

1.1 Nomenclature for Model Equations

m_2	mass of the pendulum
l_1	length of the arm
l_2	length of the pendulum
J_1	moment of inertia of the arm
J_2	moment of inertia of the pendulum about the axis of rotation through the center of mass
g	acceleration due to gravity
q_1	angel of the arm
q_2	angel of the pendulum
τ	torque on the arm

1.2 Graphic of the Structure

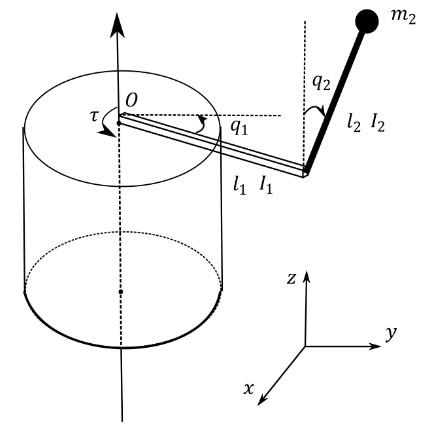

Figure 1: Structure of the Furuta Pendulum.
Source: Wang, Yang/Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme

2 Model Equations

State Vector and Input Vector:

$$\begin{aligned} \underline{x} &= (q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2)^T & &= (x_1 \ x_2 \ x_3 \ x_4)^T \\ u &= \tau \end{aligned}$$

Kinetic Energy:

$$T = \frac{1}{2}J_2 x_3^2 + \frac{1}{2}m_2[(l_1^2 + l_2^2 \sin^2 x_2)x_3^2 + l_2^2 x_4^2 + 2l_1 l_2 \cos x_2 x_3 x_4] \tag{1a}$$

(1b)

Potential Energy:

$$V = m_2 g l_2 (\cos x_2 - 1) \tag{2a}$$

(2b)

Parameters: m_2, l_1, l_2, J_1, J_2, g
Outputs: $\underline{\mathrm{x}}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the pendulum	m_2	0.2	kg
length of the arm	l_1	0.5	m
length of the pendulum	l_2	0.5	m
moment of inertia of the arm	J_1	0.02	$kg \cdot m^2$
moment of inertia of the pendulum	J_2	0.02	$kg \cdot m^2$
acceleration due to gravity	g	9.81	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

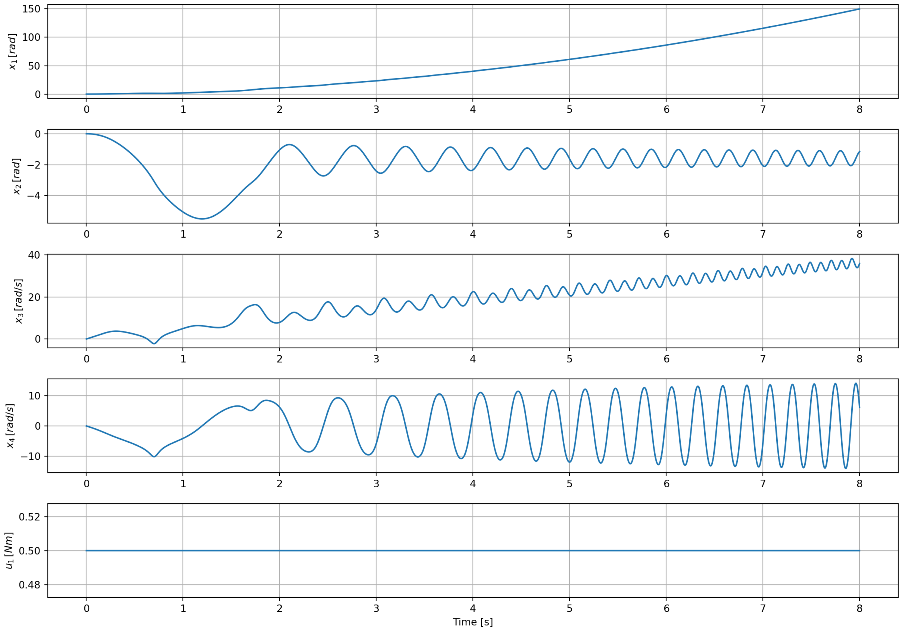

Figure 2: Simulation of the furuta pendulum.

References

[1] K. Furuta: *Swing-up control of inverted pendulum using pseudo-state feedback.*, Journal of Systems and Control Engineering, S. 263–269, published 1992.

[2] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)