Model Documentation of the Inertia Wheel Pendulum

1 Nomenclature

1.1 Nomenclature for Model Equations

m_1	mass of the pendulum
m_2	mass of the wheel
l_1	length of the pendulum
s_1	distance of the center of gravity
J_1	moment of inertia of the pendulum
J_2	moment of inertia of the wheel
g	acceleration due to gravity
τ	torque
q_1	angle between the vertical rest position and the pendulum
q_2	angel of the wheel

1.2 Graphic of the Structure

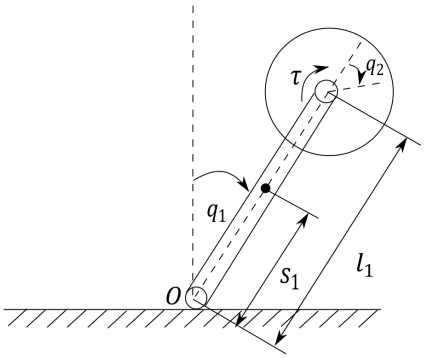

Figure 1: Structure of the IWP.
Source: Wang, Yang/Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (q_1\ q_2\ \dot{q}_1\ \dot{q}_2)^T \qquad = (x_1\ x_2\ x_3\ x_4)^T$$
$$u = \tau$$

System Equations:

$$\dot{x}_1 = x_3 \tag{1a}$$
$$\dot{x}_2 = x_4 \tag{1b}$$
$$\dot{x}_3 = \frac{J_2(m_1 s_1 + m_2 l_1) g \sin x_1 - J_2 u}{J_2(m_1 s_1^2 + m_2 l_1^2 + J_1)} \tag{1c}$$
$$\dot{x}_4 = \frac{-J_2(m_1 s_1 + m_2 l_1) g \sin x_1 + (m_1 s_1^2 + m_2 l_1^2 + J_1 + J_2) u}{J_2(m_1 s_1^2 + m_2 l_1^2 + J_1)} \tag{1d}$$

Parameters: m_1, m_2, l_1, s_1, J_1, J_2, g
Outputs: $\underline{x}$

2.1 Assumptions

1. The dissipative forces are not taken into account.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the pendulum	m_1	0.5	kg
mass of the wheel	m_2	1	kg
length of the pendulum	l_1	0.5	m
distance of the center of gravity	s_1	0.25	m
moment of inertia of the pendulum	J_1	0.02	$kg \cdot m^2$
moment of inertia of the wheel	J_2	0.002	$kg \cdot m^2$
acceleration due to gravity	g	10	$\frac{m}{s^2}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.
Kinetic Energy:

$$T = \frac{1}{2}(m_1 s_1^2 + m_2 l_1^2 + J_1 + J_2) x_3^2 + J_2 x_3 x_4 + \frac{1}{2} J_2 x_4^2 \tag{2}$$

Potential Energy:

$$V = (m_1 s_1 - m_2 l_1) g (\cos x_1 - 1) \tag{3}$$

4 Simulation

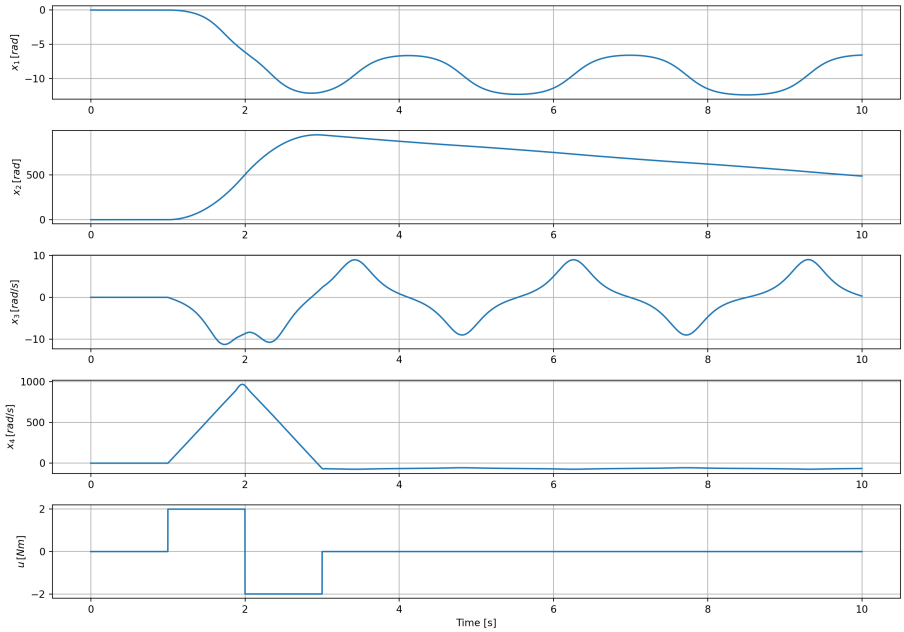

Figure 2: Simulation of the iwp.

References

[1] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)