Model Documentation of the Pendubot

1 Nomenclature

1.1 Nomenclature for Model Equations

s_i	distance from the joint to the center of gravity of link i, where $i = 1, 2$
m_i	mass of the link i, where $i = 1, 2$
J_i	moment of inertia of the link i, where $i = 1, 2$
l_1	length of link 1
τ	torque
q_1	angle between basis and link 1
p_1	angle between link 1 and link 2

1.2 Graphic of the Structure

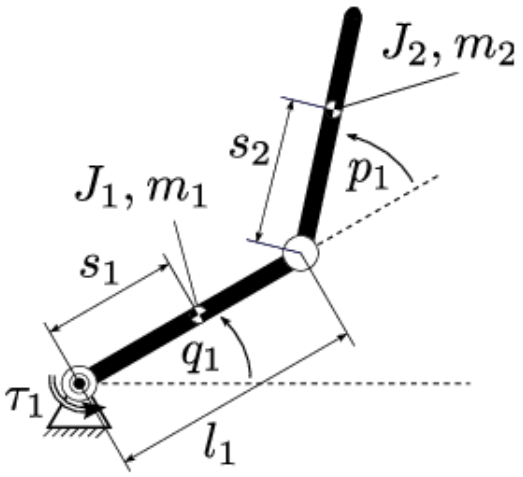

Figure 1: Structure of the Pendubot.
Source: Knoll, Carsten/ Betrachtetes System: unteraktuierten Zweigelenkmanipulator.

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (p_1\ q_1\ \dot{p}_1\ \dot{q}_1)^T \qquad = (x_1\ x_2\ x_3\ x_4)^T$$
$$u = \tau$$

Kinetic Energy:

$$\begin{aligned} T_{trans} &= \frac{1}{2} m_1 s_1^2 x_3^2 \sin^2 x_1 + \frac{1}{2} m_1 s_1^2 x_3^2 \cos^2 x_1 \\ &+ \frac{m_2}{2} (-l_1 x_3 \sin x_1 - s_2 (x_3 + x_4) \sin(x_1 + x_2))^2 \\ &+ \frac{m_2}{2} (l_1 x_3 \cos x_1 + s_2 (x_3 + x_4) \cos(x_1 + x_2))^2 \\ T_{rot} &= \frac{1}{2} J_1 x_3^2 + \frac{1}{2} J_2 (x_3 + x_4)^2 \end{aligned}$$

Potential Energy:

$$V = 0$$

Parameters: s_1, s_2, m_1, m_2, J_1, J_2, l_1
Outputs: $\underline{x}$

2.1 Assumptions

1. Friction is not taken into account.

2.2 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
distance from the joint to the center of gravity of link 1	s_1	0.1	m
distance from the joint to the center of gravity of link 2	s_2	0.25	m
mass of link 1	m_1	0.5	kg
mass of link 2	m_2	0.6	kg
moment of inertia of link 1	J_1	0.002	$kg \cdot m^2$
moment of inertia of link 2	J_2	0.001	$kg \cdot m^2$
length of link 1	l_1	0.2	m

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

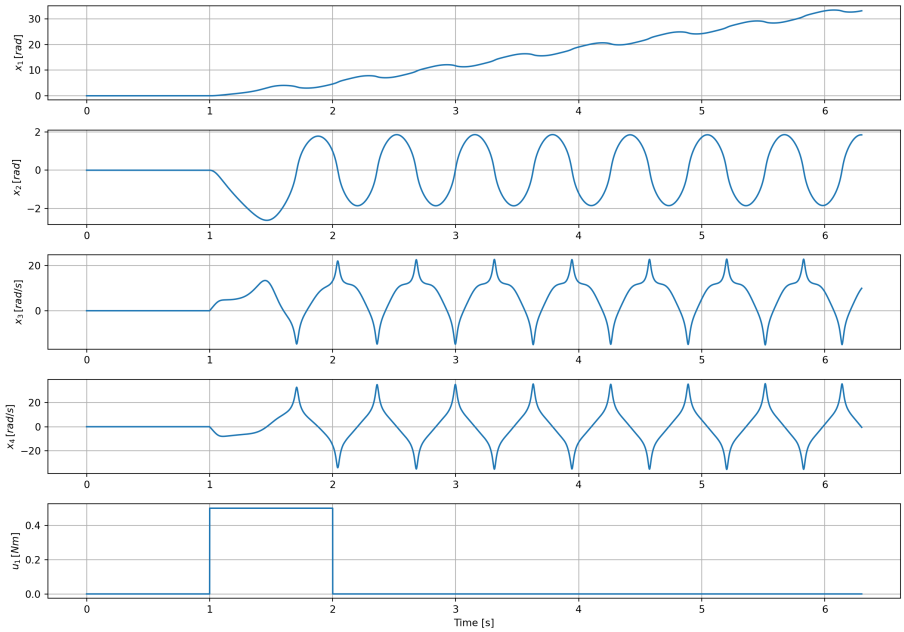

Figure 2: Simulation of the pendubot.

References

[1] Knoll, Carsten: *Betrachtetes System: unteraktuierten Zweigelenkmanipulator.*, Jupyter Notebook published 2016.
`https://github.com/cknoll/beispiele/blob/master/zweigelenk_manipulator.ipynb`

[2] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)