Model Documentation of the Translational Oscillator with Rotational Actuator (TORA)

1 Nomenclature

1.1 Nomenclature for Model Equations

m_1	mass of the cart
m_2	mass of the pendulum
l_1	length of the pendulum
J_1	moment of inertia of the pendulum
α	spring constant
τ	torque
q_1	position of the cart
q_2	angel of the pendulum

1.2 Graphic of the Structure

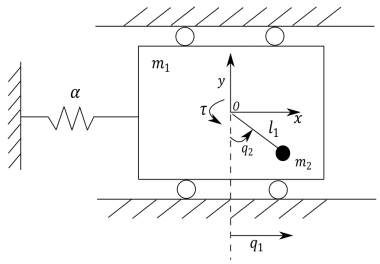

Figure 1: Structure of the TORA.
Source: Wang, Yang/Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme

2 Model Equations

State Vector and Input Vector:

$$\underline{x} = (q_1\ q_2\ \dot{q}_1\ \dot{q}_2)^T \qquad = (x_1\ x_2\ x_3\ x_4)^T$$
$$u = \tau$$

System Equations:

$$\dot{x}_1 = x_3 \tag{1a}$$
$$\dot{x}_2 = x_4 \tag{1b}$$
$$\dot{x}_3 = \frac{(m_2 l_1^2 + J_1)(-\alpha x_1 + m_2 l_1 x_4^2 \sin x_2) - m_2 l_1 \cos x_2 u}{(m_1 + m_2)(m_2 l_1^2 + J_1) - m_2^2 l_1^2 \cos^2 x_2} \tag{1c}$$
$$\dot{x}_4 = \frac{-m_2 l_1 \cos x_2 (-\alpha x_1 + m_2 l_1 x_4^2 \sin x_2) + (m_1 + m_2) u}{(m_1 + m_2)(m_2 l_1^2 + J_1) - m_2^2 l_1^2 \cos^2 x_2} \tag{1d}$$

Parameters: m_1, m_2, l_1, J_1, α
Outputs: $\underline{x}$

2.1 Exemplary parameter values

Parameter Name	Symbol	Value	Unit
mass of the cart	m_1	5	kg
mass of the pendulum	m_2	0.8	kg
length of the pendulum	l_1	0.5	m
moment of inertia of the pendulum	J_1	0.2	$kg \cdot m^2$
spring constant	a	3	$\frac{N}{m}$

3 Derivation and Explanation

The Lagrangian mechanics was used for the solution.

4 Simulation

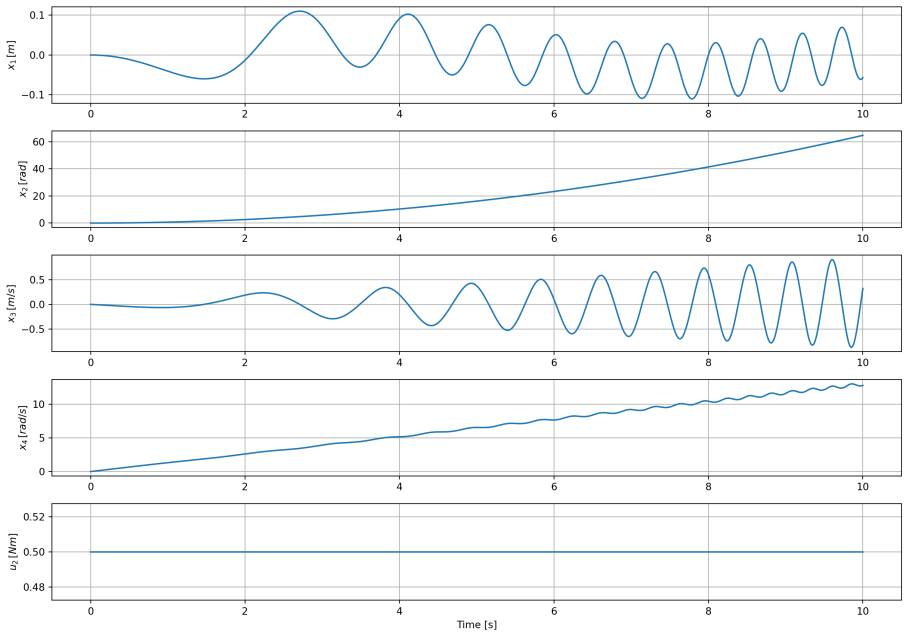

Figure 2: Simulation of the tora.

References

[1] C.-J. Wan, D. Bernstein und V. Coppola: *Global stabilization of the oscillating eccentric rotor.* In: *Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on*, Bd. 4, S. 4024–4029 vol.4, Dec 1994.

[2] Wang, Yang: *Erstellung eines regelungstheoretischen Katalogs unteraktuierter mechanischer Systeme*, master thesis at the Institut of Control Theory TU Dresden, published 2016.
(not publicly accessible)