Model Documentation of the 1D Wave Equation

1 Nomenclature

1.1 Nomenclature for Model Equations

t	time
z	space
c	propagation speed
$u(z,t)$	input function
$x(z,t)$	wanted function describing the material transport

2 Model Equations

System Equations:

$$\begin{aligned} \frac{1}{c^2}\ddot{x}(z,t) - x''(z,t) &= 0 && z \in (0,l], t > 0 \\ x(0,t) &= 0 && t > 0 \\ \dot{x}(l,t) &= u(t) && t > 0 \\ x(z,0) &= 0 && z \in [0,l] \\ x'(z,0) &= 0 && z \in [0,l] \end{aligned}$$

Parameters: c

2.1 Exemplary parameter values

Parameter Name	Symbol	Value
propagation speed of the wave	c	1

3 Derivation and Explanation

Weak form:

$$0 = \frac{1}{c^2}\int_{z=0}^{z=l} \ddot{x}(z,t)\varphi(z)\,dz - \int_{z=0}^{z=l} x''(z,t)\varphi(z)\,dz$$

with partial integration

$$0 = \frac{1}{c^2}\int_0^l \ddot{x}\varphi\,dz - [x'\varphi]_0^l + \int_0^l x'\varphi'\,dz$$

$$0 = \frac{1}{c^2}\int_0^l \ddot{x}\varphi\,dz - u(t)\varphi(l) + x'(0,t)\varphi(0) + \int_0^l x'\varphi'\,dz$$

4 Simulation

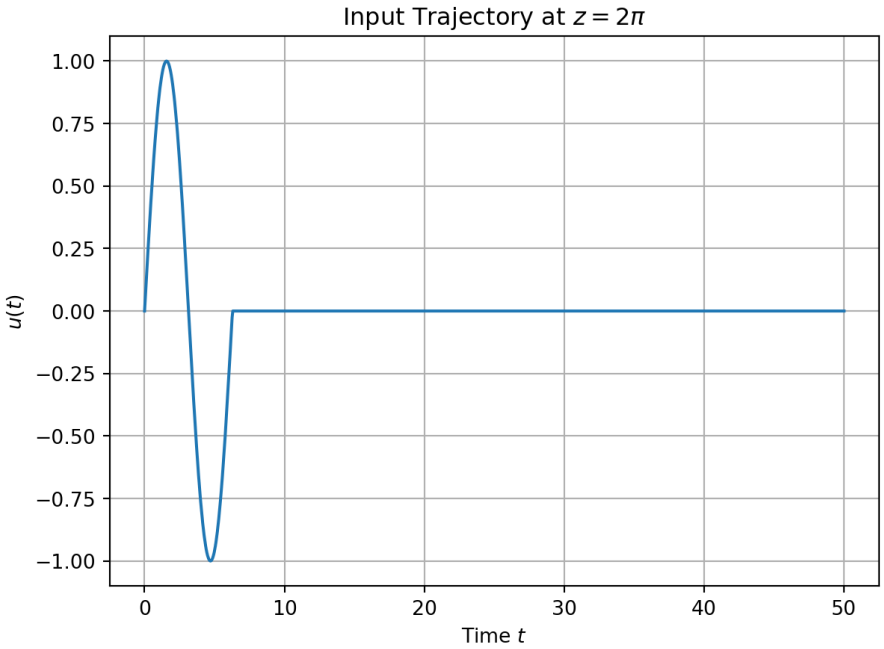

Figure 1: Simulation of the Wave Equation 1D.

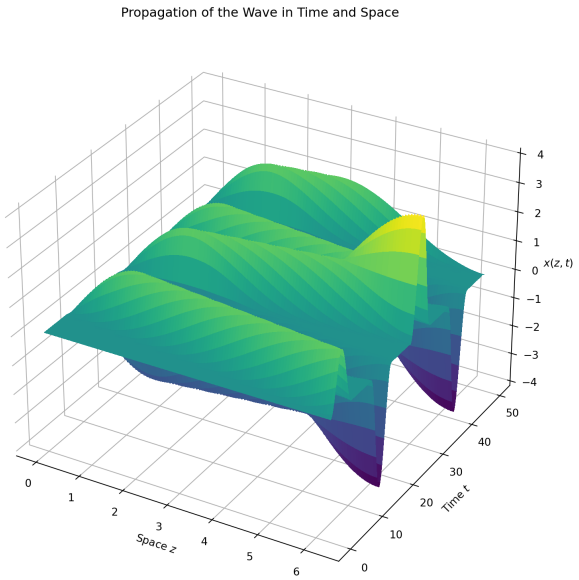

Figure 2: Simulation of the Wave Equation 1D.

References

[1] Stefan Ecklebe, Marcus Riesmeier:
https://pyinduct.readthedocs.io/en/master/readme.html